Phil 162 / Math 162 // Winter Quarter 2018 // Stanford University

PHILOSOPHY OF MATHEMATICS

Class: Monday, Wednesday, 1:30-2:50pm // 200-303

Instructor: Dr. Silvia De Toffoli // silviadt@stanford.edu
Office: 90-92I // Office Hours: Tuesday 3-4pm
Teaching Assistant: Declan Thompson // declan@stanford.edu
Office: 90-92E // Office Hours: Thursday, Friday, 11:00-12:00

DESCRIPTION

Mathematics is a very peculiar human activity. It delivers a type of knowledge that is particularly stable, often conceived as *a priori* and necessary. Moreover, this knowledge is about abstract entities, which seem to have no connection to us, spatio-temporal creatures, and yet it plays a crucial role in our scientific endeavors. Many philosophical questions emerge naturally: What is the nature of mathematical objects? How can we learn anything about them? Where does the stability of mathematics comes from? What is the significance of results showing the limits of such knowledge, such as Gödel's incompleteness theorem? The first part of the course will survey traditional approaches to philosophy of mathematics ("the big Isms") and consider the viability of their answers to some of the previous questions: logicism, intuitionism, formalism, and Hilbert's program. Moreover, arguments for and against mathematical platonism will be considered. The second part will focus on philosophical issues emerging from the actual practice of mathematics. We will tackle questions such as: What does it mean to prove in mathematics? Why do mathematicians re-prove the same theorems? What is the role of visualization and diagrammatic reasoning in mathematics? To conclude, we will explore the aesthetic dimension of mathematics, focusing on mathematical beauty.

PREREQUISITES

A basic knowledge of logic is necessary. The students should have taken at least PHIL 150, or PHIL 50. Moreover, PHIL 80 is recommended. Background in mathematics will be useful, but it is not required.

TEXTS

Øystein Linnebo, *Philosophy of Mathematics*, Princeton University Press (2017). Other materials will be available on Canvas.

OPTIONAL

- Benacerraf and Putnam, *Philosophy of Mathematics: Selected Readings*, Cambridge University Press (2nd edition, 1984)
- Mancosu (ed.), *The Philosophy of Mathematical Practice*, Oxford University Press (2008)
- Shapiro, *Thinking about Mathematics*, Oxford University Press (2000)

REQUIREMENTS

Students should do the assigned readings and participate actively in class. Every week they will send a short (from a paragraph to a page) response to the assigned readings for one of the classes. Such response is due 24 hours before the beginning of the class (they will be able to choose whether to reply to Monday's or to Wednesday's readings and accordingly to send their responses on Sunday or Tuesday). Students will write a midterm paper (5-6 pages) and have the option of writing a final paper (5-6 pages) or completing a final exam. Grading will be based on correctness *and* clarity. The final grade will be so divided:

- Participation: 10%
- Reading responses: 15%
- 1st paper: 35%
- 2nd paper or written exam: 40%

POLICIES

Late papers will receive grade penalties.

Incompletes are not granted. Exceptions to these policies will be made in the case of medical or family emergency.

Students with Documented Disabilities: Students who may need an academic accommodation based on the impact of a disability must initiate the request with the Office of Accessible Education (OAE). Professional staff will evaluate the request with required documentation, recommend reasonable accommodations, and prepare an Accommodation Letter for faculty. Unless the student has a temporary disability, Accommodation letters are issued for the entire academic year. Students should contact the OAE as soon as possible since timely notice is needed to coordinate accommodations. The OAE is located at 563 Salvatierra Walk, phone: 723-1066, URL: https://oae.stanford.edu/.

SCHEDULE

Week 1: The Nature of Mathematics

Jan 8 // Introduction

Jan 10 // Linnebo, Ch. 1

Week 2: Number Systems and Geometries

Jan 15 // Martin Luther King, Jr. Day. NO CLASS

Jan 17 // Fernando Q. Gouvêa, "From Numbers to Number Systems" and Gray, "Geometry" in: Gowers (ed.), *Princeton Companion to Mathematics* (2008)

Week 3: Logicism + Notations

Jan 22 // Linnebo, Ch. 2
Gottlob Frege, *The Foundations of Arithmetic* (1884), §55-91, 106-109

Jan 24 // Frege, *Begriffsschrift* (1879), Preface
Danielle Macbeth, "Seeing How It Goes: Paper-and-Pencil Reasoning in Mathematical Practice" (2012)

Week 4: Formalism + Hilbert's Progam

Jan 29 // Linnebo, Ch. 3

Jan 31 // Linnebo, Ch. 4, until §4.6
David Hilbert, "On the Infinite" (1926)

Week 5: Limitative Results + Intuitionism

Feb 5 // Guest Lecture by Declan Thompson – Linnebo, §4.6
Ernest Nagel and James R. Newman, *Gödel's Proof* (2001), Ch. 1, 6-7

Feb 7 // Linnebo, Ch. 5
Arend Heyting, "Disputation" (1956)

→ 1st PAPER DUE (Feb 10)

Week 6: Indispensability Argument + Benacerraf's Dilemma

Feb 12 // Linnebo, Ch. 6

Feb 14 // Linnebo, pp. 102-104
Paul Benacerraf, "Mathematical Truth" (1973)

Week 7: Nominalism

Feb 19 // Presidents' Day – NO CLASS

Feb 21 // Linnebo, Ch. 7
Hartry Field, "Realism and Anti-Realism about Mathematics" in: *Realism, Mathematics, and Modality* (1989)

Week 8: Proofs

Feb 26 // Willian Thurston, "Proof and Progress in Mathematics" (1994)

Feb 28 // Guest Lecture by Rebecca Morris – "Motivated Proofs" manuscript

Week 9: Visualization

Mar 5 // Marcus Giaquinto, "Visualizing in Mathematics" in: *The Philosophy of Mathematical Practic*e (2008)

Mar 7 // Silvia De Toffoli, "'Chasing' the Diagram – the Use of Visualizations in Algebraic Reasoning" (2017)

Week 10: Euclidean Diagrams + Mathematical Cognition

Mar 12 // Guest Lecture by John Mumma – Mumma, "Proofs, pictures, and Euclid" (2010)
Suggested: Kenneth Manders, "The Euclidean Diagram" (1995) in: *The Philosophy of Mathematical Practic*e (2008)

Mar 14 // Helen De Cruz and Johan De Smedt, "Mathematical Symbols as Epistemic Actions" (2010)

→ 2st PAPER DUE or WRITTEN EXAM (March 21) the exam will be in 200-303 from 3:30pm to 6:30pm