

Silvia De Toffoli

Curriculum Vitae

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Philosophy Department,
Princeton University
Marx Hall, Room 208
Princeton, NY 085

Specialization: Philosophy of Mathematics, Epistemology

Competence: Philosophy of Science, Logic, Nietzsche, Philosophy and Literature

employment

PRINCETON UNIVERSITY

2019 – Postdoctoral Research Associate and Lecturer, Philosophy

education

STANFORD UNIVERSITY

2019 **PhD, Philosophy**

Dissertation: *Epistemic Roles of Mathematical Diagrams*

Committee: Thomas Ryckman (advisor), Marcus Giaquinto, Michael Friedman, Krista Lawlor

As visiting student: Columbia University (fall 2016), University of Pennsylvania (fall 2017)

Goethe University Frankfurt (spring 2018)

2019 **PhD Minor, Philosophy, Literature, and the Arts** (2019)

Advisor: R. Lanier Anderson

2016 MA, philosophy

TECHNICAL UNIVERSITY OF BERLIN

2013 **PhD, Mathematics**

Dissertation: *Degrees of Essentiality for Secants of Knots*

Advisor: John M. Sullivan

UNIVERSITY OF BOLOGNA

2009 MS, Mathematics (110/110 *summa cum laude*)

Dissertation: *Different Flavors of the Poincaré Homology Sphere*

Advisors: Luca Migliorini (Bologna) and Patrick Popescu-Pampu (Paris 7)

As visiting student: University of Paris7 (2007-2008)

2006 BS, Mathematics (110/110 *summa cum laude*)

As visiting student: University of Calgary (2004-2005)

publications

JOURNAL ARTICLES

- 2017 “‘Chasing’ The Diagram – The Use of Visualizations in Algebraic Reasoning,” *The Review of Symbolic Logic*, Volume 10, Number 1, 158-186.
- *Nominated for inclusion in the *Philosopher’s Annual*, as one of the ten best philosophy papers published in 2017.
- 2014 “Forms and Roles of Diagrams in Knot Theory” (with V. Giardino), *Erkenntnis*, Volume 79, 829-842.

ARTICLES IN BOOKS

- 2019 “Leopardi: ‘Everything is Evil,’” in: *Evil—A History*, Ed: A. P. Chignell. Oxford University Press, 350-357.
- 2016 “Envisioning Transformations – The Practice of Topology” (with V. Giardino), in: *Mathematical Cultures*, Ed: B. Larvor. Birkhäuser, 25-50.
- 2015 “An Inquiry into the Practice of Proving in Low-Dimensional Topology” (with V. Giardino) In: *From Logic to Practice*, Eds: G. Lolli, M. Panza, G. Venturi. *Boston Studies in the Philosophy and History of Science*, Volume 308, 315-336.

EDITORIAL WORK

- 2018 Special issue of *Endeavour* (with P. Findlen and G. Priest) With a **co-authored introduction**: “Tools of Reason: The Practice of Scientific Diagramming from Antiquity to the Present” *Endeavour*, Volume 42 (2-3), 49–59.

OTHERS

- 2012 “Einführung in Meta-Vermeer: Experimentelle Studien über originale und manipulierte Bilder” (with Y. Sakamoto), in: *Das Licht im Zeitalter von Rembrandt und Vermeer*, Ein Handbuch der Forschungsgruppe Historische Lichtgefüge, Jovis, 50-53.
- 2012 “Meta-Vermeer: A Topological Reinterpretation of a Masterpiece” (with Y. Sakamoto), *Proceedings of Bridges 2012: Mathematics, Music, Art, Architecture, Culture*, 499-502, 2012.
- 2008 “Numeri. Divagazioni, calcoli, giochi” (with D. De Toffoli, D. Zaccariotto) (introduction by Stefano Bartezzaghi), *Stampa Alternativa*, Viterbo. 2017: 2nd Edition (preface by Furio Honsell), Kangourou editions.

grants and awards

- 2017-19 Ric Weiland Graduate Fellow in the Humanities & Sciences, Stanford University
- 2017 France-Stanford Visiting Student Researcher Fellowship (IHPST, Institute for History and Philosophy of Sciences and Technology), spring 2017, sponsor: Marco Panza, CNRS, Paris
- 2013 Certificate of Distinction from *the Berlin Mathematical School* for the Ph.D. thesis “Degrees of essentiality for secants of knots”

- 2007-08 *Erasmus* scholarship, to study for one academic year at the University of Paris 7
- 2005-06 Grant for academic achievements from the University of Bologna (“Incentivi studenti terzo anno, per merito”)
- 2004-05 TASSEP (Transatlantic Science Student Exchange Program) scholarship, to study for one academic year at University of Calgary, Canada

teaching

AS PRIMARY INSTRUCTOR

- *Theories of Knowledge*, PHIL 313, Fall Semester 2020-2021, Princeton University
- *Philosophy of Mathematics* (with John Burgess), PHIL 314, Fall Semester 2019-2020, Princeton University
- *Philosophy of Mathematics*, PHIL 162, MATH 162, (TA: Declan Thompson) Winter Quarter 2017-2018, Stanford University
- *Introduction to Philosophy*, PHIL 102, Fall Semester 2016-2017, City College New York
- *Algebraic Topology*, MA 512, Spring Semester 2013-2014, Montana State University, Bozeman

AS TEACHING ASSISTANT

- *Philosophy and Literature*, PHIL 81, (Instructors: R. Lanier Anderson and Joshua Landy) Winter Quarter 2016-2017, Stanford University
- *Epistemology*, PHIL 184, (Instructor: Eli Alshanetsky) Spring Quarter 2015-2016, Stanford University
- *Introduction to Philosophy of Science*, PHIL 60, (Instructor: Helen Longino) Winter Quarter 2015-2016, Stanford University
- *Topology* (Instructor: John Sullivan), Berlin Mathematical School Master course, Winter Semester 2010-2011, Technical University of Berlin

conference and symposia organization

- With John Burgess: *The ‘End’ of Philosophy of Mathematics?*, Conference at Princeton University (3-5 April 2020)

PAST

- *Association for the Philosophy of Mathematical Practice* symposium, 16th Congress of Logic, Methodology and Philosophy of Science and Technology, Prague (5-10 August 2019), <http://clmpst2019.flu.cas.cz>
- *Association for the Philosophy of Mathematical Practice* symposium, XXIV World Congress of Philosophy, Beijing (13-20 August 2018)
- With Rebecca Morris: *Mathematical Reasoning*, Workshop at Stanford University (9-10 February 2018)

- *Varieties of Visualization in Mathematics*, Workshop as part of the Fourth International Meeting of the Association for the Philosophy of Mathematical Practice, Salvador, Brazil (23-27 October 2017)
- With Paula Findlen and Greg Priest: **Tools of Reason: The Practice of Scientific Diagramming from Antiquity to the Present**, Workshop at Stanford University (10-11 February 2017)

professional service

- 2019 – Organizing the **Princeton Philosophy of Mathematics Reading Group**
- 2019 – **PhilPapers Editor** of the categories “**Mathematical Practice**,” “**Visualization in Mathematics**,” “**Philosophy of Literature**,” and “**Fiction**.”
- 2018 – **Member of the Directive Committee** of the *Association for the Philosophy of Mathematical Practice* (APMP)

REFEREEING

Synthese, Erkenntnis, Journal for General Philosophy of Science, Journal of Humanistic Mathematics, Manuscripta, Theoria, Dialectica, Éditions de la Sorbonne

PROGRAM COMMITTEE FOR CONFERENCES

- 2020 *Diagrams 2020*, 11th International Conference on the Theory and Application of Diagrams, Philosophy Track, Tallinn (24-28 August)
- 2020 [member of the organizing Committee] *5th International Conference of the Association for the Philosophy of Mathematical Practice*, ETH, Zurich (18-21 January)
- 2018 *Diagrams 2018*, 10th International Conference on the Theory and Application of Diagrams, Philosophy Track, Edinburgh (18-22 June)

presentations

Mathematics of weaving, Humboldt University, Berlin (June 5, 2020)

TBD

University of Trento, joint talk department of philosophy and mathematics (May 25, 2020)

TBD

Dimensions of Epistemic Justification, University of Cologne (March 26-27, 2020)

Commentator at Large

Understanding Mathematical Explanation: Uniting Philosophical and Educational Perspectives, Rutgers University (April 6, 2020)

TBD

The 'End' of Philosophy of Mathematics?, Princeton University (April 3, 2020)

"Mathematical Justification Ain't What it Seems"

5th International Conference of the Association for the Philosophy of Mathematical Practice, ETH, Zurich (January 18-21, 2020)

"A Fallibilist Account of Mathematical Justification"

2019

- Carnegie Mellon University, philosophy department, colloquium talk (8 November 2019)
“Diagrammatic Notations in Mathematical Proofs”
- Rutgers University, Talk at the *Proof Comprehension Research Group* (1 November 2019)
“Mathematical Diagrams”
- 16th Congress of Logic, Methodology and Philosophy of Science and Technology, Prague (5-10 August)
“The Epistemic Basing Relation in Mathematics”
- Masterclass in the *Philosophy of Mathematical Practice*, Vrije Universiteit Brussel (19-21 June)
“Mathematical Justification in Practice”
- Explanatory and Heuristic Power of Mathematics* (invited paper), Roma La Sapienza (13-14 June)
“Grasping How a Proof Supports its Conclusion”
- Intersem Talk*, organized by Mic Detlefsen, Paris 7, Sphere, (6 June)
“Rigorous Diagrammatic Proofs”
- Big Proofs* Workshop, International Centre for Mathematical Sciences—ICMS (invited paper), Edinburgh (26-31 May)
“Diagrammatic Notations in Mathematical Proofs”
- Macalester College, department colloquium (April 25)
“Mathematical Justification in Practice”
- Pacific APA*, Society for Realist-Antirealist Discussion, Vancouver (April 17-20)
“Proof, Simil-Proofs, and Mathematical Justification”

2018

- Mathematics in Action* (invited paper), Henri Poincaré Archives, Nancy (13-14 December)
“Mathematical Justification in Practice”
- Association for the Philosophy of Mathematical Practice* session, XXIV World Congress of Philosophy, Beijing (13-20 August)
“What is a Mathematical Diagram”
- XXIV World Congress of Philosophy, Beijing (13-20 August)
“Fruitful Over-determination for Mathematical Notations”
- Proofs and Representations* Workshop (invited paper), Munich Center for Mathematical Philosophy (6-8 July)
“Heterogenous Notations for Mathematical Proofs”
- Diagrams 2018*, 10th International Conference on the Theory and Application of Diagrams, Philosophy Track (18-22 June)
“Fruitful Over-Determination in Knot Diagrams”
- Stanford University, Psychology Department, Jay McClelland Laboratory (6 March)
“Diagrams in Mathematics”
- University of British Columbia, Okanagan campus, department colloquium (23 February)
“Diagrams in Mathematical Practice”
- Eastern APA*, Society for Realist-Antirealist Discussion, Savannah (3-7 January)
“Proofs and Diagrams in Mathematics”

2017

- 4th International Conference of the *Association for the Philosophy of Mathematical Practice*, *Varieties of Visualization* Workshop, Salvador, Brazil (23-27 October)
“Varieties of Mathematical Diagrams”

Visual Reasoning and Intuition in Mathematics. From Kant's Euclidicity to Digitalization conference (invited paper), Free University of Berlin, (29 June-1 July)

“Knots and Embeddings: New Tools for Incongruent Counterparts”

Logic in Question 7 conference (invited paper), Paris Sorbonne (13-14 June)

“Thinking with Diagrams: The Case of Mathematics”

PhilMath seminar series (invited paper), IHPST, Paris (12 June)

“Varieties of Mathematical Diagrams”

PROOFS workshop (invited paper), IHPST, Paris (1-2 June)

“The Notion of Proof in the Practice of Mathematics”

Mathématiques en action seminar series, Archives Henri Poincaré, Nancy (24 April)

“What is a Mathematical Diagram?”

Tools of Reason workshop, Stanford University (10-11 February)

“Thinking with Diagrams: The Case of Mathematics”

2016

Midwest PhilMath Workshop 17, Notre Dame University (12-13 November)

“Thinking with Diagrams: The Case of Mathematics”

City College New York, department colloquium (22 September)

“Aspects of Diagrammatic Reasoning in Mathematics”

Vrije Universiteit Brussel, colloquium (08 July)

“Aspects of Diagrammatic Reasoning in Category Theory” (with I. Goyvaerts)

University of Udine, mathematics and computer science colloquium (29 June)

“Pensare con i Diagrammi: il caso dei Nodi”

Five Years MCMP: Quo Vadis, Mathematical Philosophy? conference (invited paper), Munich Center for Mathematical Philosophy (2-4 June)

“Varieties of Visualizations in Mathematics”

43rd philosophy of science conference in Dubrovnik (invited paper), Croatia (11-15 April)

“Diagrams in Algebra”

2015

3rd International Conference of the Association for the Philosophy of Mathematical Practice (keynote speaker), Institut Henri Poincaré, Paris (2-4 November)

“‘Chasing’ The Diagram – The Use of Visualizations in Algebraic Reasoning”

Proofs and Pictures conference, University of South Carolina, Columbia (20 March)

“‘Diagram Chase’ Techniques – The Use of Space in Algebraic Reasoning”

2009–2014

What is? Seminar, Berlin Mathematical School (8 February 2013)

“What is a Rational Tangle?”

Filosofia della matematica: dalla logica alla pratica conference, Scuola Normale Superiore, Pisa, Italy (24-36 September 2012)

“La dinamica dei diagrammi in teoria dei nodi” (with V. Giardino)

Società Italiana di Filosofia Analitica (SIFA) conference, Alghero, Italy (13-15 September 2012)

“Roles and Forms of Diagrams in Knot Theory” (with V. Giardino)

Mathematical Cultures conference, London (10-13 September 2012)

“Low-dimensional topology as visual mathematics” (with V. Giardino)

Free University of Berlin, Sybille Krämer's colloquium (19 June 2012)
"Die epistemische Rollen von Diagrammen in der Topologie"
Société de philosophie analytique (SOPHA) conference, École Normale Supérieure, Paris (5 May 2012)
"Diagrammes et Illustrations en Topologie: Le Cas des Nœuds"
Work in Progress Seminar, Munich Center of Mathematical Philosophy, Munich (19 January 2012)
"The Knot Moves: A Case Study on Diagram Dynamics"
University of Sevilla, philosophy colloquium (15 November 2011)
"Use and Epistemic Roles of Diagrams in Knot Theory"
What is? Seminar, Berlin Mathematical School (4 December 2009)
"What is a Seifert Surface?"

languages

Italian (mother tongue), English (excellent), French and German (fluent reading and speaking)

references

Thomas Ryckman, Professor of Philosophy, Stanford University

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Building 100, Room 101A
450 Serra Mall
Stanford, CA 9430

Marcus Giaquinto, Honorary Emeritus Professor of Philosophy, University College London

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London N19 3TR, UK

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450 Serra Mall
Stanford, CA 94305

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Building 100, Room 101L
450 Serra Mall
Stanford, CA 94305

Helen Longino (teaching), Clarence Irving Lewis Professor of Philosophy, Stanford University

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Building 90, Room 92J
450 Serra Mall
Stanford, CA 94305

John M. Sullivan (mathematics), Professor of Mathematics, Technical University of Berlin

sullivan@math.tu-berlin.de
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TU Berlin, Room MA 802
Straße des 17. Juni 136
10623 Berlin, Germany

work in progress: philosophy of mathematics and epistemology

RIGOROUS MATHEMATICAL DIAGRAMS

Although traditionally neglected, diagrams have recently attracted attention from philosophers of mathematics. In this paper, I provide a general framework for isolating the characteristic traits of mathematical diagrams and evaluating their epistemic roles – especially in contemporary practice. I present various paradigmatic examples of diagrams and claim that they are dynamic representations that we use to think, and that they differ in philosophically important ways from *illustrations*, which are static and not constrained. In particular, I focus on the systematic use of diagrams and their two-dimensional layout to propose a technical definition which carves out the phenomenon of most interest to scholars working on mathematical diagrams. I argue that our naïve notion of *diagram* includes two distinct properties that are better kept separate: roughly, (a) the break with the linearity of written natural languages, and (b) the visual aspect typical of pictures (which is due to the relevance of geometric and topological properties). I argue that in some (rigorous) proofs, diagrams play an essential role and present two examples of such proofs from different branches of contemporary mathematics.

MATHEMATICAL JUSTIFICATION: GROUNDWORK FOR A FALLIBILIST ACCOUNT

In this paper, I present some of the central norms for doxastic justification that are at play in contemporary mathematical practice. Although such norms are about individual agents, they have an important social component. Moreover, what is required for being justified changes according to the social role the subject is playing. Whereas for the laywoman pure testimony is enough and for the clairvoyant the reliability of her super-power would suffice, for the expert

mathematician a *mathematical argument* is needed. Such an argument is what I label a *simil-proof* (SP) – that is, an argument that looks like a proof to the relevant expert(s).

To “grasp” a SP involves understanding how it supports its conclusion and also being able to *share* it in the appropriate milieu. Thus, I contend, *being justified* in professional mathematical contexts requires *the ability to justify* – often in a public context. One striking respect in which my account of mathematical (doxastic) justification differs from more traditional ones is that it has a fallibilist component: justification can come apart from knowledge, even in mathematics, since a subject may be justified in believing a false proposition or in believing a true proposition by improperly grasping a SP that is in fact fallacious.

MANIPULATIVE IMAGINATION: THE CASE OF KNOT DIAGRAMS

Diagrams are an essential part of many proofs. This does not mean that it would be impossible to prove the same result without diagrams, but it does mean that without them, in some cases, we would compromise our ability to *grasp* the proof.

When we use diagrams, we recruit cognitive abilities that were first developed by manipulating concrete objects and then redeploy them in the abstract domain. Here I focus on knot diagrams, and the way in which they support rigorous diagrammatic reasoning. When a practitioner bases her belief about a thesis concerning knots on a proof involving diagrams, she is not relying (in my view) on an untrained form of intuition alone. The cognitive ability at play is a form of *enhanced manipulative imagination* which involves intuition of two- and three-dimensional space as well as certain motor abilities, and which can be enhanced by training. Moreover, this type of imagination can be connected to a foundational apparatus linking knot diagrams to mathematical knots (via the Reidemeister theorem).

By presenting various examples I will argue that this specific type of intuition has a proper role in the justification of mathematical propositions in and possibly beyond knot theory.

THE GRAPHICAL NOTATION FOR MONOIDAL CATEGORIES (Invited for a special issue of *Synthese*), with I. Goyvaerts (Vrije University Brussel), responsible for the mathematical details.

The focus of this article is the use of diagrams in category theory. In this domain, the different notations that are widely adopted in practice bridge algebraic and geometric reasoning. In fact, category theory is at once an abstract mathematical field, where the reasoning is algebraic in nature, and a field in which it is possible to exploit topological intuition through a well-defined graphical notation. I introduce two kinds of notations, which are related by duality: the algebraic and the graphical. Although both of these notations are two-dimensional and considered diagrammatic, the graphical notation alone enables mathematicians to deploy topological intuition. These diagrammatic notations can be interpreted as cognitive instruments, used by practitioners not only as heuristic aids, but as authentic tools of reason. They are not only deployed to arrive at new conjectures and discoveries, but are also used to prove results rigorously, thereby playing genuine epistemic roles. The rigor of arguments in the graphical notation is guaranteed by soundness and completeness theorems.

RIGOR AND INTUITION: TWO MISPLACED WORRIES (Under Revise & Resubmit)

Criteria of acceptability for mathematical proofs are field-dependent. In topology, though not in most other domains, it is sometimes acceptable to appeal to visual intuition to support inferential steps. In previous co-authored papers (De Toffoli and Giardino 2014, 2015, 2016) we aimed to spell out how topological proofs work on their own terms, without appealing to formal proofs that might be associated with them. In this single-author paper, I address two criticisms that have been raised against our approach: (1) that it leads to a form of relativism according to which validity is equated with social agreement, and (2) that it implies an anti-formalizability thesis according to which it is not the case that all rigorous mathematical proofs can be formalized. I reject both criticisms and suggest that our previous case studies provide insight into the plausibility of two related but quite different theses.

work in progress: history of philosophy

KANT'S INCONGRUENT COUNTERPARTS

In this paper, I introduce some mathematical tools for thinking about incongruent counterparts. Kant famously employed them to prove different (and even contradictory) statements about the nature of space: in particular, to support Newton's conception of absolute space, and then to support his transcendental idealism.

Recent contemporary literature on incongruent counterparts has almost exclusively focused on two notions: non-orientable spaces and four-dimensional spaces. My aims are to broaden the framework in which to think about incongruent counterparts and to suggest new ways in which we can raise questions about them. To do so, I use modern mathematical ideas and visualizations. One central concept, which I also think motivates all four of Kant's arguments involving incongruent counterparts, is that of *embedding*. By considering the example of *mathematical knots*, I will explain what embeddings are and consider the roles they play with respect to incongruent counterparts.

By considering certain key practices in mathematics – especially conventional elements such as the choice of orientation of a manifold – we can recover Kant's point about the impossibility of distinguishing a right hand from a left hand by concepts alone, and reinterpret it in a contemporary mathematical setting.

THE USEFUL MYTH OF LOGIC: NIETZSCHE AS A FICTIONALIST

Nietzsche's bitter attacks on logic present interpretative challenges, but they are under-discussed in the literature. Perhaps this is due to their seeming implausibility. The first goal of this article is to suggest the opposite by showing that in fact they reveal a deep philosophical insight.

For Nietzsche, logic is, on the one hand, a truth-oriented practice of careful reasoning and, on the other hand, a formal discipline in the Aristotelian tradition – one that studies inference

patterns and conceptual representations. In this article, I articulate how the relationship between careful reasoning and formal logic can be understood in the context of Nietzsche's *perspectivism*. I also consider his claim that it is the "optimism" of logic that leads us astray in thinking that the perspectives we create and operate within are "already there," so to speak, in an objective world. In doing so, I develop a new interpretation of Nietzsche as a *logical fictionalist*.