

# MATHEMATICAL RIGOR

## BETWEEN THEORY AND PRACTICE

**John P. Burgess & Silvia De Toffoli**

*with guest appearances by*

Benjamin Morison, Desmond Hogan, Jeremy Avigad, Zoe Ashton, Neil Barton, Alan Baker, and William D'Alessandro

### COURSE DESCRIPTION

Over the last century, the philosophy of mathematics has been primarily focused on the quest to establish logical foundations and to justify axioms. But there are also philosophical issues that have to do with how living, breathing human agents practice mathematics. Our seminar will explore the middle ground between the tidy abstract realm of mathematical theories and the messy concrete world of human mathematicians. While the correctness of a mathematical argument is nowadays often identified with its formalizability, traditional proofs frequently present gaps and involve various high-level inferences; they include diagrams as well as heterogeneous notations: in short, they are unlike formal proofs. What, then, is the relationship between formal and informal proofs? How can we explain our reliability in getting things right?

"Gray ... is all theory, and green the golden tree of life."

**\*34.42.**  $\vdash (z). R^t z = P^t Q^t z . \supset . R = P \upharpoonright Q$   
*Dem.*  
 $\vdash . *14.21 . \supset \vdash : Hp . \supset (z) . E ! R^t z : (z) . E ! P^t Q^t z \quad (1)$   
 $\vdash . (1) . *34.41 . \supset \vdash : Hp . \supset (z) . R^t z = (P \upharpoonright Q)^t z :$   
 $[*30.42.(1)] \quad \supset : R = P \upharpoonright Q : . \supset \vdash . Prop$

**\*34.5.**  $\vdash : xR^2 y . \equiv . (\exists z) . xRz . zRy \quad [*34.1 . (*34.02)]$

**\*34.51.**  $\vdash : xR^2 y . \equiv . (\exists z, w) . xRz . zRw . wRy$   
*Dem.*  
 $\vdash . *34.1 . (*34.03) . \supset$   
 $\vdash : . xR^2 y . \equiv : (\exists w) . xR^2 w . wRy :$   
 $[*34.5] \quad \equiv : (\exists w) : (\exists z) . xRz . zRw : wRy :$   
 $[*11.55] \quad \equiv : (\exists w, z) . xRz . zRw . wRy :$   
 $[*11.2] \quad \equiv : (\exists z, w) . xRz . zRw . wRy : . \supset \vdash . Prop$

**\*34.52.**  $\vdash . R^3 = R \upharpoonright R^2 \quad [*34.21]$

**\*34.53.**  $\vdash : \exists ! R^2 . \equiv . \exists ! D^t R \cap \Gamma^t R \quad [*34.3]$

**\*34.531.**  $\vdash : D^t R \cap \Gamma^t R = \Lambda . \equiv . R^2 = \Lambda \quad [*34.53 . Transp]$

**\*34.54.**  $\vdash : xRx . \supset . xR^2 x$   
*Dem.*  
 $\vdash . *4.24 . \supset \vdash : xRx . \supset . xRx . xRx .$   
 $[*10.24] \quad \supset . (\exists y) . xRy . yRx .$   
 $[*34.5] \quad \supset . xR^2 x : \supset \vdash . Prop$

**\*34.55.**  $\vdash : R^2 \in S . \equiv : xRy . yRz . \supset_{x,y,z} . xSz \quad [*34.5 . *10.23]$

**\*34.56.**  $\vdash . D^t R^2 \subset D^t R . \Gamma^t R^2 \subset \Gamma^t R . C^t R^2 \subset C^t R \quad [*34.36.38]$



from Russell & Whitehead *Principia Mathematica*

Fields Medalist Maryam Mirzakhani lecturing

TENTATIVE SCHEDULE  
TUESDAYS 10:00-12:50 / 201 MARX HALL

- WEEK 1 (SEP 07)      POSING THE QUESTION  
Burgess *Rigor & Structure*, chapters 1 & 2  
Steiner *Mathematical Knowledge*, chapter 3, section 3  
*optional* Burgess "Proofs about Proofs" and "Foundational Work"  
*optional* Avigad "Does Philosophy Still Need Mathematics and Vice Versa?"
- WEEK 2 (SEP 14)      HISTORICAL BACKGROUND: EUCLID  
\*\*\*Guest Appearance: **Benjamin Morison**\*\*\*  
Beere/Morison "A Mathematical Form of Knowing How: the Nature of Problems in Euclid's Geometry"  
Sections §§3,4,8,9 are optional.
- WEEK 3 (SEP 21)      HISTORICAL BACKGROUND: KANT  
\*\*\*Guest Appearance: **Desmond Hogan**\*\*\*  
Hogan "Kant and the Character of Mathematical Inference"
- WEEK 4 (SEP 28)      HISTORICAL BACKGROUND: EULER  
Lakatos *Proofs and Refutations*, chapter 1, sections 1-5 and 9  
Steiner *Mathematical Knowledge*, chapter 3, section 4
- WEEK 5 (OCT 05)      FORMAL vs INFORMAL PROOFS: A DEBATE AMONG PHILOSOPHERS, PART I  
\*\*\*Guest Appearance: **Jeremy Avigad**\*\*\*  
Avigad "Reliability of Mathematical Inference"  
Tanswell "A Problem with the Dependence of Informal Proofs on Formal Proofs"  
*optional* Azzouni "The Derivation-Indicator View of Mathematical Practice"
- WEEK 6 (OCT 12)      FORMAL vs INFORMAL PROOFS: A DEBATE AMONG PHILOSOPHERS, PART II  
Fallis "Intentional Gaps in Mathematical Proofs"  
Andersen "Acceptable Gaps in Mathematical Proofs"
- WEEK 7 (OCT 26)      FORMAL vs INFORMAL PROOFS: A DEBATE AMONG MATHEMATICIANS  
Jaffe/Quinn "Theoretical Mathematics: Towards a Cultural Synthesis..."  
Thurston "On Proof and Progress in Mathematics"  
*optional* Replies to Jaffe/Quinn and Jaffe/Quinn replies
- WEEK 8 (NOV 02)      PROOFS AND JUSTIFICATION: A DEBATE AMONG MATHEMATICIANS  
Zeilberger "THEOREMS FOR A PRICE: Tomorrow's Semi-Rigorous Mathematical Culture"  
Hales "Formal Proofs"
- WEEK 9 (NOV 09)      PROOFS AND JUSTIFICATION: A DEBATE AMONG PHILOSOPHERS  
Paseau "What is the Point of Complete Rigour?"  
De Toffoli "Groundwork for a Fallibilist Account of Mathematics"  
*optional* Pelc "Why Do We Believe Theorems?"  
*optional* Rav "Why Do We Prove Theorems?"

WEEK 10 (NOV 16) PROOFS AND JUSTIFICATION: A DEBATE AMONG PHILOSOPHERS, PART II

\*\*\*Guest Appearances: **Neil Barton** and **Zoe Ashton**\*\*\*

Barton "Mathematical Gettier Cases and Their Implications" (manuscript)

Ashton "Each Another's Audience: A Rhetorical Account of Rigor" (manuscript)

WEEK 11 (NOV 23) PROOFS, JUSTIFICATIONS, AND DIAGRAMS

De Toffoli "Successful Transmission of Justification Across Fallacious Arguments" (manuscript)

De Toffoli "Who is Afraid of Mathematical Diagrams?" (manuscript)

*optional* De Toffoli and Fontanari "Recalcitrant Disagreement in Mathematics: An Endless and Depressing Controversy in the History of Italian Algebraic Geometry"

WEEK 12 (NOV 30) NON-DEDUCTIVE METHODS IN MATHEMATICS

\*\*\*Guest Appearance: **Alan Baker** and **William D'Alessandro**\*\*\*

Baker "Is There a Problem of Induction for Mathematics?"

*optional* Fallis "The Epistemic Status of Probabilistic Proofs"

COURSE MECHANICS

GRADUATE STUDENTS	UNDERGRADUATE STUDENTS
<p>Princeton philosophy graduate students seeking to earn a unit in connection with the seminar may write three short (1500± words) papers each related to one week's session, containing a clear and concise account of the main questions raised and the most important points about them made in the readings and discussion, plus a tentative conclusion favoring one or another position, or indicating further issues that would need to be explored.</p> <p>Papers will be due by 5 pm Princeton time            08 OCT (last class day before undergrad midterms)            23 NOV (last day before Thanksgiving break)            14 DEC ("dean's date")</p> <p>For one paper a student may substitute undertaking the oral presentation of one main reading for a given week, and leading / moderating the ensuing discussion, as per the department's oral presentation requirement.</p> <p>If the student prefers, a single longer (3000± words) paper may be substituted for the last two papers, to be due 14 DEC with a brief preliminary draft due 23 NOV.</p>	<p>The Office of the Dean of the College prohibits undergraduates officially enrolling in Philosophy graduate courses, but the ban can be worked around as follows:</p> <p>The interest student should obtain from their academic dean a form petitioning for a reading course that just happens to have the same instructors, readings, and meeting times as the graduate seminar.</p> <p>The form will need to be signed by JPB (as instructor &amp; director of undergraduate studies). The form also calls for the department chair's signature, but JPB can take care of that.</p> <p>The form must then be submitted to the student's dean together with this syllabus highlighting the following statement about <b>grading</b>:</p> <p><b>Undergraduates will graded based on the three papers as described under requirements for graduate students. Class participation will also be taken into account.</b></p>