

## PHILOSOPHY OF MATHEMATICS

**Class:** Monday, Wednesday, 1:30-2:50pm // 200-303

**Instructor:** Dr. Silvia De Toffoli // [silviadt@stanford.edu](mailto:silviadt@stanford.edu)

Office: 90-92I // Office Hours: Tuesday 3-4pm

**Teaching Assistant:** Declan Thompson // [declan@stanford.edu](mailto:declan@stanford.edu)

Office: 90-92E // Office Hours: Thursday, Friday, 11:00-12:00pm

### [DESCRIPTION]

Mathematics is a very peculiar human activity. It delivers a type of knowledge that is particularly stable, often conceived as *a priori* and necessary. Moreover, this knowledge is about abstract entities, which seem to have no connection to us, spatio-temporal creatures, and yet it plays a crucial role in our scientific endeavors. Many philosophical questions emerge naturally: What is the nature of mathematical objects? How can we learn anything about them? Where does the stability of mathematics come from? What is the significance of results showing the limits of such knowledge, such as Gödel's incompleteness theorem? The first part of the course will survey traditional approaches to philosophy of mathematics ("the big Isms") and consider the viability of their answers to some of the previous questions: logicism, intuitionism, formalism, and Hilbert's program. Moreover, arguments for and against mathematical platonism will be considered. The second part will focus on philosophical issues emerging from the actual practice of mathematics. We will tackle questions such as: What does it mean to prove in mathematics? Why do mathematicians re-prove the same theorems? What is the role of visualization and diagrammatic reasoning in mathematics? To conclude, we will explore the aesthetic dimension of mathematics, focusing on mathematical beauty.

### [PREREQUISITES]

A basic knowledge of logic is necessary. The students should have taken at least PHIL 150, or PHIL 50. Moreover, PHIL 80 is recommended. Background in mathematics will be useful, but it is not required.

### [TEXTS]

Linnebo, *Philosophy of Mathematics*, Princeton University Press (2017)

Other materials will be on Canvas.

### OPTIONAL

- Benacerraf and Putnam, *Philosophy of Mathematics: Selected Readings*, Cambridge University Press (2<sup>nd</sup> edition, 1984)
- Mancosu (ed.), *The Philosophy of Mathematical Practice*, Oxford University Press (2008)
- Shapiro, *Thinking about Mathematics*, Oxford University Press (2000)

## [REQUIREMENTS]

Students should do the assigned readings and participate actively in class. Every week they will send a short (from a paragraph to a page) response to the assigned readings for one of the classes. Such response is due 24 hours before the beginning of the class (they will be able to choose whether to reply to Monday's or to Wednesday's readings and accordingly to send their responses on Sunday or Tuesday). Students will write a midterm paper (5-6 pages) and have the option of writing a final paper (5-6 pages) or completing a final exam. Grading will be based on correctness *and* clarity. The final grade will be so divided:

- Participation: 10%
- Reading responses: 15%
- 1<sup>st</sup> paper: 35%
- 2<sup>nd</sup> paper or written exam: 40%

## [POLICIES]

Late papers will receive grade penalties.

Incompletes are not granted. Exceptions to these policies will be made in the case of medical or family emergency.

Students with Documented Disabilities: Students who may need an academic accommodation based on the impact of a disability must initiate the request with the Office of Accessible Education (OAE). Professional staff will evaluate the request with required documentation, recommend reasonable accommodations, and prepare an Accommodation Letter for faculty. Unless the student has a temporary disability, Accommodation letters are issued for the entire academic year. Students should contact the OAE as soon as possible since timely notice is needed to coordinate accommodations. The OAE is located at 563 Salvatierra Walk, phone: 723-1066, URL: <https://oae.stanford.edu/>.

## [SCHEDULE]

### **Week 1: The Nature of Mathematics**

Jan 8 // Introduction

Jan 10 // Linnebo, Ch. 1

### **Week 2: Number Systems and Geometries**

Jan 15 // Martin Luther King, Jr. Day. NO CLASS

Jan 17 // Gouvêa, "From Numbers to Number Systems" and Gray, "Geometry" in: Gowers (ed.), *Princeton Companion to Mathematics* (2008)

### **Week 3: Logicism + Notations**

Jan 22 // Linnebo, Ch. 2

Frege, *The Foundations of Arithmetic* (1884), §55-91, 106-109

Jan 24 // Frege, *Begriffsschrift* (1879), Preface

Macbeth, "Seeing How It Goes: Paper-and-Pencil Reasoning in Mathematical Practice" (2012)

### **Week 4: Formalism + Hilbert's Program**

Jan 29 // Linnebo, Ch. 3

Jan 31 // Linnebo, Ch. 4, until §4.6

Hilbert, "On the Infinite" (1926)

### **Week 5: Limitative Results + Intuitionism**

Feb 5 // Guest Lecture by Declan Thompson – Linnebo, §4.6

Nagel and Newman, *Gödel's Proof* (2001), Ch. 1, 6-7

Feb 7 // Linnebo, Ch. 5

Heyting, "Disputation" (1956)

Suggested: Heyting, "The intuitionistic foundations of mathematics" (1931), pp. 52-56

→ 1<sup>st</sup> PAPER DUE (Feb 10)

### **Week 6: Indispensability Argument + Benacerraf's Dilemma**

Feb 12 // Linnebo, Ch. 6

Feb 14 // Linnebo, pp. 102-104

Benacerraf, "Mathematical Truth" (1973)

### **Week 7: Nominalism**

Feb 19 // Presidents' Day – NO CLASS

Feb 21 // Linnebo, Ch. 7

Field, "Realism and Anti-Realism about Mathematics" in: *Realism, Mathematics, and Modality* (1989)

**Week 8: Proofs**

Feb 26 // Thurston, "Proof and Progress in Mathematics" (1994)

Feb 28 // Guest Lecture by Becky Morris – "Motivated Proofs" manuscript

**Week 9: Visualization**

Mar 5 // Giaquinto, "Visualizing in Mathematics" in: *The Philosophy of Mathematical Practice* (2008)

Mar 7 // De Toffoli, "'Chasing' the Diagram – the Use of Visualizations in Algebraic Reasoning"  
(2017)

**Week 10: Euclidean Diagrams + Beauty**

Mar 12 // Guest Lecture by John Mumma – Mumma, "Proofs, pictures, and Euclid" (2010)

Suggested: Manders, "The Euclidean Diagram" (1995) in: *The Philosophy of Mathematical Practice* (2008)

Mar 14 // Rota, "Phenomenology of Mathematical Beauty" (1997)

→ 2<sup>st</sup> PAPER DUE or WRITTEN EXAM (March 21) the exam will be in 200-303 from 3:30pm to 6:30pm